19. **(a)** Displacement: $s(5)-s(0)=12-2=10$ m

(b) Average velocity $=\dfrac{10 \text{ m}}{5 \text{ sec}}=2$ m/sec

(c) Velocity $=s'(t)=2t-3$
At $t=4$,
velocity $=s'(4)=2(4)-3=5$ m/sec

(d) Acceleration $=s''(t)=2$ m/sec^2

(e) The particle changes direction when
$s'(t)=2t-3=0$, so $t=\dfrac{3}{2}$ sec.

(f) Since the acceleration is always positive, the position s is at a minimum when the particle changes direction, at $t=\dfrac{3}{2}$ sec. Its position at this time is $s\left(\dfrac{3}{2}\right)=-\dfrac{1}{4}$ m.

20. **(a)** $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dt}(-t^3+7t^2-14t+8)$
$v(t)=-3t^2+14t-14$

(b) $a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}(-3t^2+14t-14)$
$a(t)=-6t+14$

(c) $v(t)=-3t^2+14t-14=0$
$t\approx 1.451,\ 3.215$

(d) The particle starts at the point $s=8$ when $t=0$ and moves left until it stops at $s=-0.631$ when $t=1.451$, then it moves right to the point $s=2.113$ when $t=3.215$ where it stops again, and finally continues left from there on.

21. **(a)**
$$\begin{aligned} v(t)&=\frac{ds}{dt}\\ &=\frac{d}{dt}[(t-2)^2(t-4)]\\ &=(t-2)^2(1)+(t-4)\cdot 2(t-2)\\ &=(t-2)[(t-2)+2(t-4)]\\ &=(t-2)(3t-10)\end{aligned}$$

(b) $a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}[(t-2)(3t-10)]$
$a(t)=6t-16$

(c) $v(t)=(t-2)(3t-10)=0$
$t=2,\dfrac{10}{3}$

(d) The particle starts at the point $s=-16$ when $t=0$ and move right until it stops at $s=0$ when $t=2$, then it moves left to the point $s=-1.185$ when $t=\dfrac{10}{3}$ where it stops again, and finally continues right from there on.

22. **(a)** $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dt}(t^3-6t^2+8t+2)$
$v(t)=3t^2-12t+8$

(b) $a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}(3t^2-12t+8)$
$a(t)=6t-12$

(c) $v(t)=3t^2-12t+8=0$
$t\approx 0.845,\ 3.155$

(d) The particle starts at the point $s=2$ when $t=0$ and moves right until it stops at $s=5.079$ when $t=0.845$, then it moves left to the point $s=-1.079$ when $t=3.155$ where it stops again, and finally continues right from there on.

23. $v(t)=s'(t)=3t^2-12t+9$
$a(t)=v'(t)=6t-12$
Find when velocity is zero.
$3t^2-12t+9=0$
$3(t^2-4t+3)=0$
$3(t-1)(t-3)=0$
$t=1$ or $t=3$
At $t=1$, the acceleration is $a(1)=-6$ m/sec^2
At $t=3$, the acceleration is $a(3)=6$ m/sec^2

24. $a(t)=v'(t)=6t^2-18t+12$
Find when acceleration is zero.
$6t^2-18t+12=0$
$6(t^2-3t+2)=0$
$6(t-1)(t-2)=0$
$t=1$ or $t=2$
At $t=1$, the speed is $|v(1)|=|0|=0$ m/sec.
At $t=2$, the speed is $|v(2)|=|-1|=1$ m/sec.

25. **(a)** $$\frac{dy}{dt} = \frac{d}{dt}\left[6\left(1-\frac{t}{12}\right)^2\right]$$

$$= \frac{d}{dt}\left[6\left(1-\frac{t}{6}+\frac{t^2}{144}\right)\right]$$

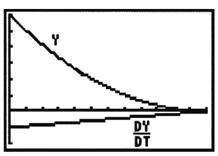
$$= \frac{d}{dt}\left(6-t+\frac{1}{24}t^2\right)$$

$$= 0-1+\frac{t}{12}$$

$$= \frac{t}{12}-1$$

(b) The fluid level is falling fastest when $\frac{dy}{dt}$ is the most negative, at $t = 0$, when $\frac{dy}{dt} = -1$. The fluid level is falling slowest at $t = 12$, when $\frac{dy}{dt} = 0$.

(c)

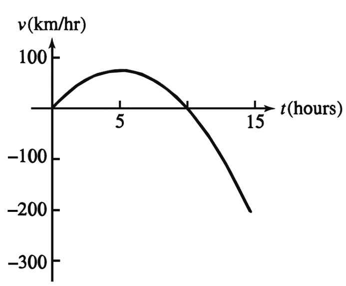


[0, 12] by [−2, 6]

y is decreasing and $\frac{dy}{dt}$ is negative over the entire interval y decreases more rapidly early in the interval, and the magnitude of $\frac{dy}{dt}$ is larger then. $\frac{dy}{dt}$ is 0 at $t = 12$, where the graph of y seems to have a horizontal tangent.

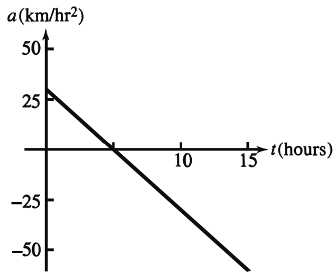
26. **(a)** To graph the velocity, we estimate the slopes at several points as follows, then connect the points to create a smooth curve.

t (hours)	0	2.5	5	7.5	10	12.5	15
v (km/hour)	0	56	75	56	0	–[illegible]4	–225

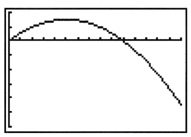


To graph the acceleration, we estimate the slope of the velocity graph at several points as follows, and then connect the points to create a smooth curve.

t (hours)	0	2.5	5	7.5	10	12.5	15
a (km/hour^2)	30	15	0	–[illegible]5	–[illegible]0	–[illegible]5	–[illegible]0

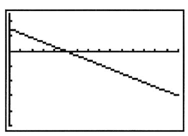


(b) $\dfrac{ds}{dt} = 30t - 3t^2$



[0, 15] by [–300, 100]

$\dfrac{d^2s}{dt^2} = 30 - 6t$



[0, 15] by [–100, 50]

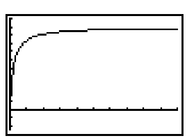
The graphs are very similar.

27. **(a)** Average cost $= \dfrac{c(100)}{100} = \dfrac{11{,}000}{100} = \$110$ per machine

(b) $c'(x) = 100 - 0.2x$
Marginal cost $= c'(100) = \$80$ per machine

(c) Actual cost of 101st machine is $c(101) - c(100) = \$79.90$, which is very close to the marginal cost calculated in part (b).

28. **(a)**



[0, 50] by [−500, 2200]

The values of x which make sense are the whole numbers, $x \geq 0$.

(b) Marginal revenue

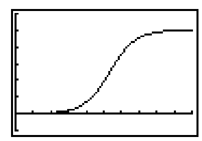
$$r'(x) = \frac{d}{dx}\left[2000\left(1 - \frac{1}{x+1}\right)\right]$$
$$= \frac{d}{dx}\left(2000 - \frac{2000}{x+1}\right)$$
$$= 0 - \frac{(x+1)(0) - (2000)(1)}{(x+1)^2}$$
$$= \frac{2000}{(x+1)^2}$$

(c) $r'(5) = \dfrac{2000}{(5+1)^2} = \dfrac{2000}{36} \approx 55.56$

The increase in revenue is approximately \$55.56.

(d) The limit is 0. This means that as x gets large, one reaches a point where very little extra revenue can be expected from selling more desks.

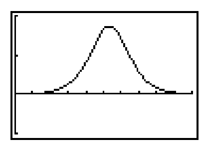
29. **(a)**



[0, 200] by [–2, 12]

(b) The values of x which make sense are the whole numbers, $x \geq 0$.

(c)



[0, 200] by [–0.1, 0.2]

P is most sensitive to changes in x when $|P'(x)|$ is largest. It is relatively sensitive to changes in x between approximately $x = 60$ and $x = 160$.

(d) The marginal profit, $P'(x)$, is greatest at $x = 106.44$. Since x must be an integer, $P(106) \approx 4.924$ thousand dollars or \$4924.

(e) $P'(50) \approx 0.013$, or \$13 per package sold
$P'(100) \approx 0.165$, or \$165 per package sold
$P'(125) \approx 0.118$, or \$118 per package sold
$P'(150) \approx 0.031$, or \$31 per package sold
$P'(175) \approx 0.006$, or \$6 per package sold
$P'(300) \approx 10^{-6}$, or \$0.001 per package sold

(f) The limit is 10. The maximum possible profit is \$10,000 monthly.

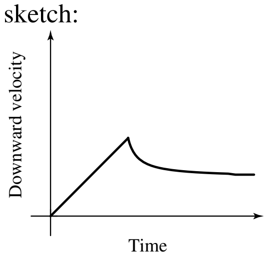
(g) Yes; in order to sell more and more packages, the company might need to lower the price to a point where they won't make any additional profit.

30. Since the particle moves along the line $y = 2$, it will be at the point (5, 2) when $x(t) = 4t^3 - 16t^2 + 15t = 5$. Use a grapher to see that this occurs when $t = 2.83$.

31. Graph C is position, graph A is velocity, and graph B is acceleration.
A is the derivative of C because it is positive, negative, and zero where C is increasing, decreasing, and has horizontal tangents, respectively. The relationship between B and A is similar.

32. Graph C is position, graph B is velocity, and graph A is acceleration.
B is the derivative of C because it is negative and zero where C is decreasing and has horizontal tangents, respectively.
A is the derivative of B because it is positive, negative, and zero where B is increasing, decreasing, and has horizontal tangents, respectively.

33. Note that "downward velocity" is positive when McCarthy is falling downward. His downward velocity increases steadily until the parachute opens, and then decreases to a constant downward velocity. One possible sketch:



34. **(a)** $\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$

When $r = 2$, $\frac{dv}{dr} = 4\pi(2)^2 = 16\pi$ cubic feet of volume per foot of radius.

(b) The increase in the volume is $\frac{4}{3}\pi(2.2)^3 - \frac{4}{3}\pi(2)^3 \approx 11.092$ cubic feet.

35. Let v_0 be the exit velocity of a particle of lava. Then $s(t) = v_0 t - 16t^2$ feet, so the velocity is $\frac{ds}{dt} = v_0 - 32t$. Solving $\frac{ds}{dt} = 0$ gives $t = \frac{v_0}{32}$. Then the maximum height, in feet, is $s\left(\frac{v_0}{32}\right) = v_0\left(\frac{v_0}{32}\right) - 16\left(\frac{v_0}{32}\right)^2 = \frac{v_0^2}{64}$. Solving $\frac{v_0^2}{64} = 1900$ gives $v_0 \approx \pm 348.712$. The exit velocity was about 348.712 ft/sec. Multiplying by $\frac{3600 \text{ sec}}{1 \text{ h}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}}$, we find that this is equivalent to about 237.758 mi/h.

36. By estimating the slope of the velocity graph at that point.

37. The motion can be simulated in parametric mode using $x_1(t) = 2t^3 - 13t^2 + 22t - 5$ and $y_1(t) = 2$ in [–6, 8] by [–3, 5].

(a) It begins at the point (–5, 2) moving in the positive direction. After a little more than one second, it has moved a bit past (6, 2) and it turns back in the negative direction for approximately 2 seconds. At the end of that time, it is near (–2, 2) and it turns back again in the positive direction. After that, it continues moving in the positive direction indefinitely, speeding up as it goes.

(b) The particle speeds up when its *speed* is increasing, which occurs during the approximate intervals $1.153 \le t \le 2.167$ and $t \ge 3.180$. It slows down during the approximate intervals $0 \le t \le 1.153$ and $2.167 \le t \le 3.180$. One way to determine the endpoints of these intervals is to use a grapher to find the minimums and maximums for the speed, $\left|\frac{dx}{dt}\right| = \left|6t^2 - 26t + 22\right|$ using function mode in the window [0, 5] by [0, 10].

(c) The particle changes direction at $t \approx 1.153$ sec and at $t \approx 3.180$ sec.

(d) The particle is at rest "instantaneously" at $t \approx 1.153$ sec and at $t \approx 3.180$ sec.

(e) The velocity starts out positive but decreasing, it becomes negative, then starts to increase, and becomes positive again and continues to increase.
The speed is decreasing, reaches 0 at $t \approx 1.15$ sec, then increases until $t \approx 2.17$ sec, decreases until $t \approx 3.18$ sec when it is 0 again, and then increases after that.

(f) The particle is at (5, 2) when $2t^3 - 13t^2 + 22t - 5 = 5$ at $t \approx 0.745$ sec, $t \approx 1.626$ sec, and at $t \approx 4.129$ sec.

38. **(a)** Solving $160 = 490t^2$ gives $t = \pm\frac{4}{7}$. It took $\frac{4}{7}$ of a second. The average velocity was $\frac{160\text{cm}}{\left(\frac{4}{7}\right)}$ sec = 280 cm/sec.

(b) $V = \frac{ds}{dt} = 980t$

$a = \frac{dV}{dt} = 980$

At $s = 160$ cm, $t = \frac{4}{7}$sec (from part (*a*)) and

$V = 980\left(\frac{4}{7}\right) = 560$ cm/sec

$a = 980$ cm/sec^2

(c) Once the balls begin falling, each flash will produce a different image. There are 16 images of the balls falling, so $\frac{16 \text{ flashes}}{\frac{4}{7} \text{ seconds}} = 28$ flashes per second.

39. Since profit = revenue – cost, the Sum and Difference Rule gives

$\frac{d}{dx}(\text{profit}) = \frac{d}{dx}(\text{revenue}) - \frac{d}{dx}(\text{cost})$, where x is the number of units produced. This means that marginal profit = marginal revenue – marginal cost.

40. False; it is the absolute value of the velocity.

41. True. The acceleration is the first derivative of the velocity which, in turn, is the second derivative of the position function.

42. C; $f'(x) = 2x + \frac{2}{x^2}$

$f'(-1) = 2(-1) + \frac{2}{(-1)^2} = 0$

43. D; $V(x) = x^3$

$\frac{dv}{dx} = 3x^2$

44. E; $\frac{ds}{dt} = \frac{d}{dt}(2 + 7t - t^2)$

$v(t) = 7 - 2t < 0$

$7 < 2t$

$\frac{7}{2} < t$

$4 > \frac{7}{2}$

45. C; $v(t) = 7 - 2t = 0$

$7 = 2t$

$t = \frac{7}{2}$

46. The growth rate is given by

$b'(t) = 10^4 - 2 \cdot 10^3(t) = 10{,}000 - 2000t.$

At $t = 0: b'(0) = 10{,}000$ bacteria/hour

At $t = 5: b'(5) = 0$ bacteria/hour

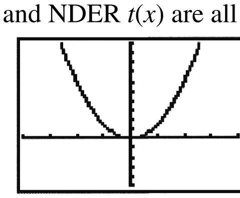
At $t = 10: b'(10) = -10{,}000$ bacteria/hour

47. **(a)** $g'(x) = \frac{d}{dx}(x^3) = 3x^2$

$h'(x) = \frac{d}{dx}(x^3 - 2) = 3x^2$

$t'(x) = \frac{d}{dx}(x^3 + 3) = 3x^2$

(b) The graphs of NDER $g(x)$, NDER $h(x)$, and NDER $t(x)$ are all the same, as shown.



[−4, 4] by [−10, 20]

(c) $f(x)$ must be of the form $f(x) = x^3 + c$, where c is a constant.

(d) Yes; $f(x) = x^3$

(e) Yes. $f(x) = x^3 + 3$

48. For $t > 0$, the speed of the aircraft in meters per second after t seconds is $v(t) = \frac{20}{9}t$.

Multiplying by $\frac{3600 \text{ sec}}{1\text{h}} \cdot \frac{1\text{km}}{1000 \text{ m}}$, we find that this is equivalent to $8t$ kilometers per hour. Solving $8t = 200$ gives $t = 25$ seconds. The aircraft takes 25 seconds to become airborne, and the distance it travels during this time is $D(25) \approx 694.444$ meters.

49. (a) Assume that f is even. Then,

$$f'(-x) = \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0} \frac{f(x-h) - f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0} \frac{f(x+k) - f(x)}{-k} = -\lim_{k\to 0} \frac{f(x+k) - f(x)}{k} = -f'(x)$$

So, f' is an odd function.

(b) Assume that f is odd. Then,

$$f'(-x) = \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0} \frac{-f(x-h) + f(x)}{h},$$

and substituting $k = -h$,

$$= \lim_{k\to 0} \frac{-f(x+k) + f(x)}{-k} = \lim_{k\to 0} \frac{f(x+k) - f(x)}{k} = f'(x)$$

So, f' is an even function.

50.
$$\frac{d}{dx}(fgh) = \frac{d}{dx}[f(gh)]$$
$$= f \cdot \frac{d}{dx}(gh) + gh \cdot \frac{d}{dx}(f)$$
$$= f\left(g \cdot \frac{dh}{dx} + h \cdot \frac{dg}{dx}\right) + gh \cdot \frac{df}{dx}$$
$$= \left(\frac{df}{dx}\right)gh + f\left(\frac{dg}{dx}\right)h + fg\left(\frac{dh}{dx}\right)$$

Section 3.5 Derivatives of Trigonometric Functions (pp. 141–147)

Exploration 1 Making a Conjecture by Graphing the Derivative

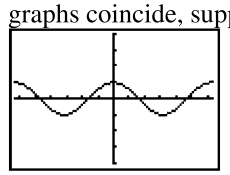
1. When the graph of $\sin x$ is increasing, the graph of $\frac{d}{dx}(\sin x)$ is positive (above the x-axis).

2. When the graph of $\sin x$ is decreasing, the graph of $\frac{d}{dx}(\sin x)$ is negative (below the x-axis).

3. When the graph of $\sin x$ stops increasing and starts decreasing, the graph of $\frac{d}{dx}(\sin x)$ crosses the x-axis from above to below.

4. The slope of the graph of $\sin x$ matches the value of $\frac{d}{dx}(\sin x)$ at these points.

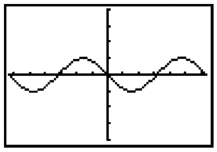
5. We conjecture that $\frac{d}{dx}(\sin x) = \cos x$. The graphs coincide, supporting our conjecture.



$[-2\pi, 2\pi]$ by $[-4, 4]$

6. When the graph of $\cos x$ is increasing, the graph of $\frac{d}{dx}(\cos x)$ is positive (above the x-axis).
When the graph of $\cos x$ is decreasing, the graph of $\frac{d}{dx}(\cos x)$ is negative (below the x-axis).
When the graph of $\cos x$ stops increasing and starts decreasing, the graph of $\frac{d}{dx}(\cos x)$ crosses the x-axis from above to below.
The slope of the graph of $\cos x$ matches the value of $\frac{d}{dx}(\cos x)$ at these points.

We conjecture that $\frac{d}{dx}(\cos x) = -\sin x$. The graphs coincide, supporting our conjecture.


$[-2\pi, 2\pi]$ by $[-4, 4]$

Quick Review 3.5

1. $135° \cdot \frac{\pi}{180°} = \frac{3\pi}{4} \approx 2.356$

2. $1.7 \cdot \frac{180°}{\pi} = \left(\frac{306}{\pi}\right)^{\circ} \approx 97.403°$

3. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

4. Domain: All reals
Range: $[-1, 1]$

5. Domain: $x \neq \frac{k\pi}{2}$ for odd integers k
Range: All reals

6. $\cos a = \pm\sqrt{1-\sin^2 a} = \pm\sqrt{1-(-1)^2} = \pm\sqrt{0} = 0$

7. If $\tan a = -1$, then $a = \frac{3\pi}{4} + k\pi$ for some integer k, so $\sin a = \pm\frac{1}{\sqrt{2}}$.

8. $$\frac{1-\cos h}{h} = \frac{(1-\cos h)(1+\cos h)}{h(1+\cos h)} = \frac{1-\cos^2 h}{h(1+\cos h)} = \frac{\sin^2 h}{h(1+\cos h)}$$

9. $y'(x) = 6x^2 - 14x$
$y'(3) = 12$
The tangent line has slope 12 and passes through (3, 1), so its equation is $y = 12(x-3)+1$, or $y = 12x - 35$.

10. $a(t) = v'(t) = 6t^2 - 14t$
$a(3) = 12$

Section 3.5 Exercises

1. $\frac{d}{dx}(1+x-\cos x) = 0+1-(-\sin x) = 1+\sin x$

2. $\frac{d}{dx}(2\sin x - \tan x) = 2\cos x - \sec^2 x$

3. $\frac{d}{dx}\left(\frac{1}{x}+5\sin x\right) = -\frac{1}{x^2}+5\cos x$

4. $$\frac{d}{dx}(x\sec x) = x\frac{d}{dx}(\sec x) + \sec x\frac{d}{dx}(x) = x\sec x\tan x + \sec x$$

5. $$\frac{d}{dx}(4-x^2\sin x) = \frac{d}{dx}(4) - \left[x^2\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(x^2)\right] = 0 - [x^2\cos x + (\sin x)(2x)] = -x^2\cos x - 2x\sin x$$

6. $$\frac{d}{dx}(3x + x\tan x) = \frac{d}{dx}(3x) + \left[x\frac{d}{dx}(\tan x) + (\tan x)\frac{d}{dx}(x)\right] = 3 + x\sec^2 x + \tan x$$

7. $\frac{d}{dx}\left(\frac{4}{\cos x}\right) = \frac{d}{dx}(4\sec x) = 4\sec x\tan x$

8. $$\frac{d}{dx}\left(\frac{x}{1+\cos x}\right) = \frac{(1+\cos x)\frac{d}{dx}(x) - x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2} = \frac{1+\cos x + x\sin x}{(1+\cos x)^2}$$

9. $\dfrac{d}{dx}\left(\dfrac{\cot x}{1+\cot x}\right)$

$= \dfrac{(1+\cot x)\frac{d}{dx}(\cot x)-(\cot x)\frac{d}{dx}(1+\cot x)}{(1+\cot x)^2}$

$= \dfrac{(1+\cot x)(-\csc^2 x)-(\cot x)(-\csc^2 x)}{(1+\cot x)^2}$

$= -\dfrac{\csc^2 x}{(1+\cot x)^2}$

$= -\dfrac{\csc^2 x \sin^2 x}{(1+\cot x)^2 \sin^2 x}$

$= -\dfrac{1}{(\sin x+\cos x)^2}$

10. $\dfrac{d}{dx}\left(\dfrac{\cos x}{1+\sin x}\right)$

$= \dfrac{(1+\sin x)\frac{d}{dx}(\cos x)-(\cos x)\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$

$= \dfrac{(1+\sin x)(-\sin x)-(\cos x)(\cos x)}{(1+\sin x)^2}$

$= \dfrac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2}$

$= \dfrac{-(1+\sin x)}{(1+\sin x)^2}$

$= -\dfrac{1}{1+\sin x}$

11. $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dx}(5\ \sin t)$

$v(t)=5\cos t$

$a(t)=\dfrac{dv}{dt}=\dfrac{d}{dx}(5\ \cos t)$

$a(t)=-5\sin t$

The weight starts at 0, goes to 5, and the oscillates between 5 and –5. The period of the motion is 2π. The speed is greatest when $\cos t=\pm 1\ (t=k\pi)$, zero when $\cos t=0\ \left(t=\dfrac{k\pi}{2},\ k \text{ odd}\right)$. The acceleration is greatest when $\sin t=\pm 1\ \left(t=\dfrac{k\pi}{2},\ k \text{ odd}\right)$, zero when $\sin t=0\ (t=k\pi)$.

12. $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dx}(7\ \cos t)$

$v(t)=-7\sin t$

$a(t)=\dfrac{dv}{dt}=\dfrac{d}{dx}(-7\ \sin t)$

$a(t)=-7\cos t$

The weight starts at 7, goes to –7, and then oscillates between –7 and 7. The period of the motion is 2π. The speed is greatest when $\sin t=\pm 1\ \left(t=\dfrac{k\pi}{2},\ k \text{ odd}\right)$, zero when $\sin t=0\ (t=k\pi)$. The acceleration is greatest when $\cos t=\pm 1\ (t=k\pi)$, zero when $\cos t=0\ \left(t=\dfrac{k\pi}{2},\ k \text{ odd}\right)$.

13. **(a)** $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dt}(2+3\sin t)$

$v(t)=3\cos t, \text{speed}=|3\cos t|$

$a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}(3\cos t)=-3\sin t$

(b) $v\left(\dfrac{\pi}{4}\right)=3\cos\left(\dfrac{\pi}{4}\right)=\dfrac{3\sqrt{2}}{2}$, speed $=\dfrac{3\sqrt{2}}{2}$

$a\left(\dfrac{\pi}{4}\right)=-3\sin\left(\dfrac{\pi}{4}\right)=-\dfrac{3\sqrt{2}}{2}$

(c) The body starts at 2, goes up to 5, goes down to –1, and then oscillates between –1 and 5. The period of motion is 2π.

14. **(a)** $v(t)=\dfrac{ds}{dt}=\dfrac{d}{dt}(1-4\cos t)$

$v(t)=4\ \sin t$, speed $=|4\ \sin t|$

$a(t)=\dfrac{dv}{dt}=\dfrac{d}{dt}(4\ \sin t)$

$a(t)=4\ \cos t$

(b) $v\left(\dfrac{\pi}{4}\right)=4\sin\left(\dfrac{\pi}{4}\right)=2\sqrt{2}$, speed $=2\sqrt{2}$

$a\left(\dfrac{\pi}{4}\right)=4\cos\left(\dfrac{\pi}{4}\right)=2\sqrt{2}$

(c) The body starts at –3, goes up to 5, and then oscillates between 5 and –3. The period of the motion is 2π.

15. **(a)** $v(t) = \frac{ds}{dt} = \frac{d}{dt}(2\sin t + 3\cos t)$

$v(t) = 2\cos t - 3\sin t,$

speed $= |2\cos t - 3\sin t|$

$a(t) = \frac{dv}{dt} = \frac{d}{dt}(2\cos t - 3\sin t)$

$a(t) = -2\sin t - 3\cos t$

(b) $v\left(\frac{\pi}{4}\right) = 2\cos\frac{\pi}{4} - 3\sin\frac{\pi}{4}$

$v\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

speed $= \frac{\sqrt{2}}{2}$

$a\left(\frac{\pi}{4}\right) = -2\sin\frac{\pi}{4} - 3\cos\frac{\pi}{4}$

$a\left(\frac{\pi}{4}\right) = \frac{-5\sqrt{2}}{2}$

(c) The body starts at 3, goes to 3.606, and then oscillates between –3.606 and 3.606. The period of the motion is 2π.

16. **(a)** $v(t) = \frac{ds}{dt} = \frac{d}{dt}(\cos t - 3\sin t)$

$v(t) = -\sin t - 3\cos t,$

speed $= |-\sin t - 3\cos t|$

$a(t) = \frac{dv}{dt} = \frac{d}{dt}(-\sin t - 3\cos t)$

$a(t) = -\cos t + 3\sin t$

(b) $v\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4} - 3\cos\frac{\pi}{4} = -2\sqrt{2}$

speed $= 2\sqrt{2}$

$a\left(\frac{\pi}{4}\right) = -\cos\frac{\pi}{4} + 3\sin\frac{\pi}{4} = \sqrt{2}$

(c) The body starts at 1, goes to –3.162, and then oscillates between 3.162 and –3.162. The period of the motion is 2π.

17. $j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}$

$f(t) = 2\cos t$

$f'(t) = -2\sin t$

$f''(t) = -2\cos t$

$f'''(t) = 2\sin t$

18. $j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}$

$f(t) = 1 + 2\cos t$

$f'(t) = -2\sin t$

$f''(t) = -2\cos t$

$f'''(t) = 2\sin t$

19. $j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}$

$f(t) = \sin t - \cos t$

$f'(t) = \cos t + \sin t$

$f''(t) = -\sin t + \cos t$

$f'''(t) = -\cos t - \sin t$

20. $j(t) = \frac{da}{dt} = \frac{d^3s}{dt}$

$f(t) = 2 + 2\sin t$

$f'(t) = 2\cos t$

$f''(t) = -2\sin t$

$f'''(t) = -2\cos t$

21. $y = \sin x + 3$

$\frac{dy}{dx} = \frac{d}{dx}(\sin x + 3) = \cos x$

$y(\pi) = \sin\pi + 3 = 3$

$y'(\pi) = \cos\pi = -1$

tangent: $y = -1(x - \pi) + 3 = -x + \pi + 3$

normal: $m_2 = -\frac{1}{m_1} = 1$

$y = (x - \pi) + 3$

22. $y = \sec x$

$\frac{dy}{dx} = \frac{d}{dx}(\sec x) = \sec x \tan x$

$y\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4} = 1.414$

$y'\left(\frac{\pi}{4}\right) = \sec\frac{\pi}{4}\tan\frac{\pi}{4} = 1.414$

tangent: $y = 1.414(x - \frac{\pi}{4}) + 1.414$

$y = 1.414x + 0.303$

normal: $m_2 = -\frac{1}{m_1} = -0.707$

$y = -0.707(x - \frac{\pi}{4}) + 1.414 = -0.707x + 1.970$

23. $y = x^2 \sin x$

$\dfrac{dy}{dx} = \dfrac{d}{dx}(x^2 \sin x) = 2x \sin x + x^2 \cos x$

$y(3) = (3)^2 \sin 3 = 1.270$

$y'(3) = 2(3)\sin 3 + (3)^2 \cos 3 = -8.063$

tangent:

$y = -8.063(x-3) + 1.270 = -8.063x + 25.460$

normal: $m_2 = -\dfrac{1}{m_1} = 0.124$

$y = 0.124(x-3) + 1.270$

$y = 0.124x + 0.898$

24.
$$\begin{aligned}
&\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} \\
&= \lim_{h\to 0} \frac{(\cos x \cos h - \sin x \sin h) - \cos x}{h} \\
&= \lim_{h\to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h} \\
&= \lim_{h\to 0}\left((\cos x)\frac{\cos h - 1}{h} - (\sin x)\frac{\sin h}{h}\right) \\
&= (\cos x)\left(\lim_{h\to 0}\frac{\cos h - 1}{h}\right) - (\sin x)\left(\lim_{h\to 0}\frac{\sin h}{h}\right) \\
&= (\cos x)(0) - (\sin x)(1) = -\sin x
\end{aligned}$$

25. **(a)**
$$\begin{aligned}
&\frac{d}{dx}\tan x \\
&= \frac{d}{dx}\frac{\sin x}{\cos x} \\
&= \frac{(\cos x)\frac{d}{dx}(\sin x) - (\sin x)\frac{d}{dx}(\cos x)}{(\cos x)^2} \\
&= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} \\
&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} \\
&= \sec^2 x
\end{aligned}$$

(b)
$$\begin{aligned}
\frac{d}{dx}\sec x &= \frac{d}{dx}\frac{1}{\cos x} \\
&= \frac{(\cos x)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\cos x)}{(\cos x)^2} \\
&= \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x} \\
&= \frac{\sin x}{\cos^2 x} \\
&= \sec x \tan x
\end{aligned}$$

26. **(a)**
$$\begin{aligned}
&\frac{d}{dx}\cot x \\
&= \frac{d}{dx}\frac{\cos x}{\sin x} \\
&= \frac{(\sin x)\frac{d}{dx}(\cos x) - (\cos x)\frac{d}{dx}(\sin x)}{(\sin x)^2} \\
&= \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x} \\
&= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} \\
&= -\frac{1}{\sin^2 x} \\
&= -\csc^2 x
\end{aligned}$$

(b)
$$\begin{aligned}
\frac{d}{dx}\csc x &= \frac{d}{dx}\frac{1}{\sin x} \\
&= \frac{(\sin x)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\sin x)}{(\sin x)^2} \\
&= \frac{(\sin x)(0) - (1)(\cos x)}{\sin^2 x} \\
&= -\frac{\cos x}{\sin^2 x} \\
&= -\csc x \cot x
\end{aligned}$$

27. $\dfrac{d}{dx}\sec x = \sec x \tan x$ which is 0 at $x = 0$, so the slope of the tangent line is 0.

$\dfrac{d}{dx}\cos x = -\sin x$ which is 0 at $x = 0$, so the slope of the tangent line is 0.

28. $\dfrac{d}{dx}\tan x = \sec^2 x = \dfrac{1}{\cos^2 x}$, which is never 0.

$\dfrac{d}{dx}\cot x = -\csc^2 x = -\dfrac{1}{\sin^2 x}$, which is never 0.

29. $y'(x) = \dfrac{d}{dx}\left(\sqrt{2}\cos x\right) = -\sqrt{2}\sin x$

$y'\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\sin\dfrac{\pi}{4} = -\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right) = -1$

The tangent line has slope -1 and passes through $\left(\dfrac{\pi}{4}, \sqrt{2}\cos\dfrac{\pi}{4}\right) = \left(\dfrac{\pi}{4}, 1\right)$, so its equation is $y = -1\left(x - \dfrac{\pi}{4}\right) + 1$, or

$y = -x + \frac{\pi}{4} + 1.$

The normal line has slope 1 and passes through $\left(\frac{\pi}{4}, 1\right)$, so its equation is $y = 1\left(x - \frac{\pi}{4}\right) + 1$, or $y = x + 1 - \frac{\pi}{4}$.

30. $y'(x) = \frac{d}{dx} \tan x = \sec^2 x$

$y'(x) = \frac{d}{dx}(2x) = 2$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

On $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the solutions are $x = \pm\frac{\pi}{4}$. The points on the curve are $\left(-\frac{\pi}{4}, -1\right)$ and $\left(\frac{\pi}{4}, 1\right)$.

31. $y'(x) = \frac{d}{dx}(4 + \cot x - 2\csc x)$

$= 0 - \csc^2 x + 2\csc x \cot x$

$= -\csc^2 x + 2\csc x \cot x$

(a) $y'\left(\frac{\pi}{2}\right) = -\csc^2 \frac{\pi}{2} + 2\csc\frac{\pi}{2}\cot\frac{\pi}{2}$

$= -1^2 + 2(1)(0)$

$= -1$

The tangent line has slope -1 and passes through $P\left(\frac{\pi}{2}, 2\right)$. Its equation is

$y = -1\left(x - \frac{\pi}{2}\right) + 2$, or $y = -x + \frac{\pi}{2} + 2$.

(b) $f'(x) = 0$

$-\csc^2 x + 2\csc x \cot x = 0$

$-\frac{1}{\sin^2 x} + \frac{2\cos x}{\sin^2 x} = 0$

$\frac{1}{\sin^2 x}(2\cos x - 1) = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

at point Q

$y\left(\frac{\pi}{3}\right) = 4 + \cot\frac{\pi}{3} - 2\csc\frac{\pi}{3}$

$= 4 + \frac{1}{\sqrt{3}} - 2\left(\frac{2}{\sqrt{3}}\right)$

$= 4 - \frac{3}{\sqrt{3}}$

$= 4 - \sqrt{3}$

The coordinates of Q are $\left(\frac{\pi}{3}, 4 - \sqrt{3}\right)$.

The equation of the horizontal line is $y = 4 - \sqrt{3}$.

32. $y'(x) = \frac{d}{dx}(1 + \sqrt{2}\csc x + \cot x)$

$= 0 + \sqrt{2}(-\csc x \cot x) + (-\csc^2 x)$

$= -\sqrt{2}\csc x \cot x - \csc^2 x$

(a) $y'\left(\frac{\pi}{4}\right) = -\sqrt{2}\csc\frac{\pi}{4}\cot\frac{\pi}{4} - \csc^2\frac{\pi}{4}$

$= -\sqrt{2}(\sqrt{2})(1) - (\sqrt{2})^2$

$= -2 - 2$

$= -4$

The tangent line has slope -4 and passes through $P\left(\frac{\pi}{4}, 4\right)$. Its equation is

$y = -4\left(x - \frac{\pi}{4}\right) + 4$, or $y = -4x + \pi + 4$.

(b) $y'(x) = 0$

$$-\sqrt{2}\csc x\cot x - \csc^2 x = 0$$
$$-\frac{\sqrt{2}\cos x}{\sin^2 x} - \frac{1}{\sin^2 x} = 0$$
$$-\frac{1}{\sin^2 x}(\sqrt{2}\cos x + 1) = 0$$
$$\cos x = -\frac{1}{\sqrt{2}}$$
$$x = \frac{3\pi}{4} \text{ at point } Q$$

$$y\left(\frac{3\pi}{4}\right) = 1 + \sqrt{2}\csc\frac{3\pi}{4} + \cot\frac{3\pi}{4}$$
$$= 1 + \sqrt{2}(\sqrt{2}) + (-1)$$
$$= 2$$

The coordinates of Q are $\left(\frac{3\pi}{4},\ 2\right)$.

The equation of the horizontal line is $y = 2$.

33. **(a)** Velocity: $s'(t) = -2\cos t$ m/sec
Speed: $|s'(t)| = |2\cos t|$ m/sec
Acceleration: $s''(t) = 2\sin t$ m/sec^2
Jerk: $s'''(t) = 2\cos t$ m/sec^3

(b) Velocity: $-2\cos\frac{\pi}{4} = -\sqrt{2}$ m/sec

Speed: $|-\sqrt{2}| = \sqrt{2}$ m/sec

Acceleration: $2\sin\frac{\pi}{4} = \sqrt{2}$ m/sec^2

Jerk: $2\cos\frac{\pi}{4} = \sqrt{2}$ m/sec^3

(c) The body starts at 2, goes to 0 and then oscillates between 0 and 4.
Speed: *Greatest* when $\cos t = \pm 1$ (or $t = k\pi$), at the center of the interval of motion.
Zero when $\cos t = 0$ (or $t = \frac{k\pi}{2}$, k odd), at the endpoints of the interval of motion.
Acceleration: *Greatest* (in magnitude) when $\sin t = \pm 1$ (or $t = \frac{k\pi}{2}$, k odd)
Zero when $\sin t = 0$ (or $t = k\pi$)
Jerk: *Greatest* (in magnitude) when $\cos t = \pm 1$ (or $t = k\pi$).
Zero when $\cos t = 0$ (or $t = \frac{k\pi}{2}$, k odd)

34. **(a)** Velocity: $s'(t) = \cos t - \sin t$ m/sec
Speed: $s'(t) = |\cos t - \sin t|$ m/sec
Acceleration: $s''(t) = -\sin t - \cos t$ m/sec^2
Jerk: $s'''(t) = -\cos t + \sin t$ m/sec^3

(b) Velocity: $\cos\frac{\pi}{4} - \sin\frac{\pi}{4} = 0$ m/sec

Speed: $|0| = 0$ m/sec

Acceleration:
$-\sin\frac{\pi}{4} - \cos\frac{\pi}{4} = -\sqrt{2}$ m/sec^2

Jerk: $-\cos\frac{\pi}{4} + \sin\frac{\pi}{4} = 0$ m/sec^3

(c) The body starts at 1, goes to $\sqrt{2}$ and then oscillates between $\pm\sqrt{2}$.
Speed:
Greatest when $t = \frac{3\pi}{4} + k\pi$

Zero when $t = \frac{\pi}{4} + k\pi$
Acceleration:
Greatest (in magnitude) when $t = \frac{\pi}{4} + k\pi$

Zero when $t = \frac{3\pi}{4} + k\pi$
Jerk:
Greatest (in magnitude) when
$t = \frac{3\pi}{4} + k\pi$

Zero when $t = \frac{\pi}{4} + k\pi$

35. $y' = \frac{d}{dx}\csc x = -\csc x\cot x$

$$y'' = \frac{d}{dx}(-\csc x\cot x)$$
$$= -(\csc x)\frac{d}{dx}(\cot x) - (\cot x)\frac{d}{dx}(\csc x)$$
$$= -(\csc x)(-\csc^2 x) - (\cot x)(-\csc x\cot x)$$
$$= \csc^3 x + \csc x\cot^2 x$$

36. $y' = \frac{d}{d\theta}(\theta \tan\theta)$

$$= \theta \frac{d}{d\theta}(\tan\theta) + (\tan\theta)\frac{d}{d\theta}(\theta)$$
$$= \theta \sec^2\theta + \tan\theta$$

$$y'' = \frac{d}{d\theta}(\theta \sec^2\theta + \tan\theta)$$
$$= \theta \frac{d}{d\theta}[(\sec\theta)(\sec\theta)] + (\sec^2\theta)\frac{d}{d\theta}(\theta) + \frac{d}{d\theta}(\tan\theta)$$
$$= \theta\left[(\sec\theta)\frac{d}{d\theta}(\sec\theta) + (\sec\theta)\frac{d}{d\theta}(\sec\theta)\right] + \sec^2\theta + \sec^2\theta$$
$$= 2\theta\sec^2\theta\tan\theta + 2\sec^2\theta$$
$$= (2\theta\tan\theta + 2)(\sec^2\theta)$$

or, writing in terms of sines and cosines,

$$= \frac{2 + 2\theta\tan\theta}{\cos^2\theta}$$
$$= \frac{2\cos\theta + 2\theta\sin\theta}{\cos^3\theta}$$

37. Continuous: Note that $g(0) = \lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \cos x = \cos(0) = 1$, and $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}(x+b) = b$. We require $\lim_{x\to 0^-} g(x) = g(0)$, so $b = 1$. The function is continuous if $b = 1$.

Differentiable: For $b = 1$, the left-hand derivative is 1 and the right-hand derivative is $-\sin(0) = 0$, so the function is not differentiable. For other values of b, the function is discontinuous at $x = 0$ and there is no left-hand derivative. So, there is no value of b that will make the function differentiable at $x = 0$.

38. Observe the pattern:

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d^5}{dx^5}\cos x = -\sin x$$
$$\frac{d^2}{dx^2}\cos x = -\cos x \qquad \frac{d^6}{dx^6}\cos x = -\cos x$$
$$\frac{d^3}{dx^3}\cos x = \sin x \qquad \frac{d^7}{dx^7}\cos x = \sin x$$
$$\frac{d^4}{dx^4}\cos x = \cos x \qquad \frac{d^8}{dx^8}\cos x = \cos x$$

Continuing the pattern, we see that

$\frac{d^n}{dx^n}\cos x = \sin x$ when $n = 4k + 3$ for any whole number k.

Since $999 = 4(249) + 3$, $\frac{d^{999}}{dx^{999}}\cos x = \sin x$.

39. Observe the pattern:

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d^5}{dx^5}\sin x = \cos x$$
$$\frac{d^2}{dx^2}\sin x = -\sin x \qquad \frac{d^6}{dx^6}\sin x = -\sin x$$
$$\frac{d^3}{dx^3}\sin x = -\cos x \qquad \frac{d^7}{dx^7}\sin x = -\cos x$$
$$\frac{d^4}{dx^4}\sin x = \sin x \qquad \frac{d^8}{dx^8}\sin x = \sin x$$

Continuing the pattern, we see that

$\frac{d^n}{dx^n}\sin x = \cos x$ when $n = 4k + 1$ for any whole number k. Since $725 = 4(181) + 1$,

$\frac{d^{725}}{dx^{725}}\sin x = \cos x$.

40. The line is tangent to the graph of $y = \sin x$ at $(0, 0)$. Since $y'(0) = \cos(0) = 1$, the line has slope 1 and its equation is $y = x$.

41. **(a)** Using $y = x$, $\sin(0.12) \approx 0.12$.

(b) $\sin(0.12) \approx 0.1197122$; the approximation is within 0.0003 of the actual value.

42. $\frac{d}{dx}\sin 2x$

$$= \frac{d}{dx}(2\sin x\cos x)$$
$$= 2\frac{d}{dx}(\sin x\cos x)$$
$$= 2\left[(\sin x)\frac{d}{dx}(\cos x) + (\cos x)\frac{d}{dx}(\sin x)\right]$$
$$= 2[(\sin x)(-\sin x) + (\cos x)(\cos x)]$$
$$= 2(\cos^2 x - \sin^2 x)$$
$$= 2\cos 2x$$

43. $\frac{d}{dx}\cos 2x$

$$= \frac{d}{dx}[(\cos x)(\cos x) - (\sin x)(\sin x)]$$
$$= \left[(\cos x)\frac{d}{dx}(\cos x) + (\cos x)\frac{d}{dx}(\cos x)\right] - \left[(\sin x)\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(\sin x)\right]$$
$$= 2(\cos x)(-\sin x) - 2(\sin x)(\cos x)$$
$$= -4\sin x\cos x$$
$$= -2(2\sin x\cos x)$$
$$= -2\sin 2x$$

44. True. $s'(t) = -3\cos t$, $s'\left(\frac{3\pi}{4}\right) = -3\cos\left(\frac{3\pi}{4}\right) = \frac{3\sqrt{2}}{2} > 0$. The derivative is positive at $t = \frac{3\pi}{4}$.

45. False. The velocity is negative and the speed is positive at $t = \frac{\pi}{4}$.

46. A; $y = \sin x + \cos x$

$$y'(x) = \cos x - \sin x$$
$$y(\pi) = \sin\pi + \cos\pi = -1$$
$$y'(\pi) = \cos\pi - \sin\pi = -1$$
$$y = -1(x - \pi) - 1$$
$$y = -x + \pi - 1$$

47. B; See 46.

$$m_2 = -\frac{1}{m_1} = -\frac{1}{-1} = 1$$
$$y = (x - \pi) - 1$$

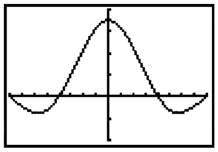
48. C; $y = x\sin x$

$$y' = \sin x + x\cos x$$
$$y'' = \cos x + \cos x - x\sin xk = -x\sin x + 2\cos x$$

49. C; $v(t) = \frac{ds}{dt} = \frac{d}{dt}(3 + \sin t)$

$$v(t) = \cos t = 0$$
$$t = \frac{\pi}{2}$$

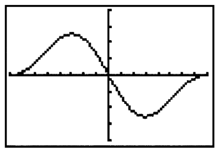
50. (a)



[–360, 360] by [–0.01, 0.02]

The limit is $\frac{\pi}{180}$ because this is the conversion factor for changing from degrees to radians.

(b)



[–360, 360] by [–0.02, 0.02]

This limit is still 0.

(c)
$$\begin{aligned}\frac{d}{dx}\sin x &= \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\\ &= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}\\ &= \lim_{h\to 0}\frac{\sin x(\cos h-1)+\cos x\sin h}{h}\\ &= \left(\lim_{h\to 0}\sin x\right)\left(\lim_{h\to 0}\frac{\cos h-1}{h}\right)+\left(\lim_{h\to 0}\cos x\right)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\\ &= (\sin x)(0)+(\cos x)\left(\frac{\pi}{180}\right)\\ &= \frac{\pi}{180}\cos x\end{aligned}$$

(d)
$$\begin{aligned}\frac{d}{dx}\cos x &= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}\\ &= \lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}\\ &= \lim_{h\to 0}\frac{(\cos x)(\cos h-1)-\sin x\sin h}{h}\\ &= \left(\lim_{h\to 0}\cos x\right)\left(\lim_{h\to 0}\frac{\cos h-1}{h}\right)-\left(\lim_{h\to 0}\sin x\right)\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\\ &= (\cos x)(0)-(\sin x)\left(\frac{\pi}{180}\right)\\ &= -\frac{\pi}{180}\sin x\end{aligned}$$

(e)
$$\begin{aligned}\frac{d^2}{dx^2}\sin x &= \frac{d}{dx}\frac{\pi}{180}\cos x\\ &= \frac{\pi}{180}\left(-\frac{\pi}{180}\sin x\right)\\ &= -\frac{\pi^2}{180^2}\sin x\end{aligned}$$

$$\frac{d^3}{dx^3}\sin x = \frac{d}{dx}\left(-\frac{\pi^2}{180^2}\sin x\right)$$
$$= -\frac{\pi^2}{180^2}\left(\frac{\pi}{180}\cos x\right)$$
$$= -\frac{\pi^3}{180^3}\cos x$$
$$\frac{d^2}{dx^2}\cos x = \frac{d}{dx}\left(-\frac{\pi}{180}\sin x\right)$$
$$= -\frac{\pi}{180}\left(\frac{\pi}{180}\cos x\right)$$
$$= -\frac{\pi^2}{180^2}\cos x$$
$$\frac{d^3}{dx^3}\cos x = \frac{d}{dx}\left(-\frac{\pi^2}{180^2}\cos x\right)$$
$$= -\frac{\pi^2}{180^2}\left(-\frac{\pi}{180}\sin x\right)$$
$$= \frac{\pi^3}{180^3}\sin x$$

51. $$\lim_{h\to 0}\frac{(\cos h - 1)}{h} = \lim_{h\to 0}\frac{(\cos h-1)(\cos h+1)}{h(\cos h+1)}$$
$$= \lim_{h\to 0}\frac{\cos^2 h - 1}{h(\cos h+1)}$$
$$= \lim_{h\to 0}\frac{-\sin^2 h}{h(\cos h+1)}$$
$$= -\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\left(\lim_{h\to 0}\frac{\sin h}{\cos h+1}\right)$$
$$= -(1)\left(\frac{0}{2}\right)$$
$$= 0$$

52. $$y' = \frac{d}{dx}(A\sin x + B\cos x) = A\cos x - B\sin x$$
$$y'' = \frac{d}{dx}(A\cos x - B\sin x)$$
$$= -A\sin x - B\cos x$$
Solve:
$$y'' - y = \sin x$$
$$(-A\sin x - B\cos x) - (A\sin x + B\cos x) = \sin x$$
$$-2A\sin x - 2B\cos x = \sin x$$
At $x = \frac{\pi}{2}$, this gives $-2A = 1$, so $A = -\frac{1}{2}$.
At $x = 0$, we have $-2B = 0$, so $B = 0$.
Thus, $A = -\frac{1}{2}$ and $B = 0$.

Quick Quiz Sections 3.4–3.5

1. $f'(1) = \frac{6-(-4)}{1-(-1)} = \frac{10}{2} = 5$; C

2. $$y' = \frac{d}{dx}(\cos x + \tan x)$$
$$= -\sin x + \sec^2 x$$
$$= -\sin x + \sec x \cdot \sec x$$
$$y'' = \frac{d}{dx}(-\sin x + \sec x\cdot \sec x)$$
$$= -\cos x + \sec x(\sec x\tan x) + \sec x(\sec x\tan x)$$
$$= -\cos x + 2\sec^2 x\tan x;\ \text{A}$$

3. $$\frac{dy}{dx} = \frac{3(2x+3)-2(3x+2)}{(2x+3)^2} = \frac{5}{(2x+3)^2};\ \text{D}$$

4. **(a)** $s(0) = -0^2 + 0 + 2 = 2$ m

(b) $$v(t) = s'(t) = \frac{d}{dt}(-t^2+t+2)$$
$$= -2t + 1 \text{ m/s}$$

(c) The particle moves to the right when $v(t) > 0$.
$-2t + 1 > 0$
$t < 0.5$
$0 \le t < 0.5$ (time must be ≥ 0)

(d) $a(t) = v'(t) = \frac{d}{dt}(-2t+1) = -2 \text{ m/s}^2$

(e) $$s(t) = 0$$
$$-t^2 + t + 2 = 0$$
$$-(t^2 - t - 2) = 0$$
$$-(t-2)(t+1) = 0$$
$t = 2$ or $t = -1$ (not in domain)
speed $= |v(2)| = |-2(2)+1| = |-3| = 3$ m/s

Chapter 3 Review Exercises
(pp. 148–151)

1. $$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 - \frac{1}{8}x^2 + \frac{1}{4}x\right) = 5x^4 - \frac{1}{4}x + \frac{1}{4}$$

2. $$\frac{dy}{dx} = \frac{d}{dx}(3 - 7x^3 + 3x^7) = -21x^2 + 21x^6$$